Chapter 15 - Design of Beams For Strength
Reading: Chapter 15 — Pages 541-576

15-1
Introduction

- This chapter will focus on the design of steel beams and wood beams.
- The beam design will only have to comply with bending stress and shear stress.
- Other beam failures will be discussed in later chapters.

Steel Beams

- Maximum bending moment of a steel beam under a given load and support arrangement is the most important information required to determine the beam size necessary to resist the flexural (bending) stress in the beam.
- Shear stress in steel beams is rarely an issue, but will still be considered.

Wood Beams

- Maximum shear stress of a wood beam under a given load and support arrangement is the most important information required to determine the beam size necessary to resist the shear stress in the beam.
- Bending moment must also be considered in wood beams.

Maximum Flexural Stress

$$\sigma_{max} = \frac{M}{S}$$

where,

σ_{max} = the maximum flexural stress due to bending

M = the maximum bending moment

S = the section modulus of the beams horizontal axis

Shear Stress

$$\tau = \frac{VQ}{It}$$

Where,

τ = the shear stress at a point in a given section of the beam

V = the shear force at the given section

Q = the first moment of the area A′ about the neutral axis, $Q = A'\bar{y}'$

A′ = the part of the areea in the cross-section above (or below) the horizontal line where the shear stress is to be calculated.

$\bar{y}'$ = the distance from the neutral axis to the centroid of the area A′

I = the moment of inertia of the entire section with respect to the neutral axis (the same I as in the flexure formula)

t = the width of the cross-section at the horizontal line where the shear stress is being calculated

Beam problems are of two major types: analysis and design

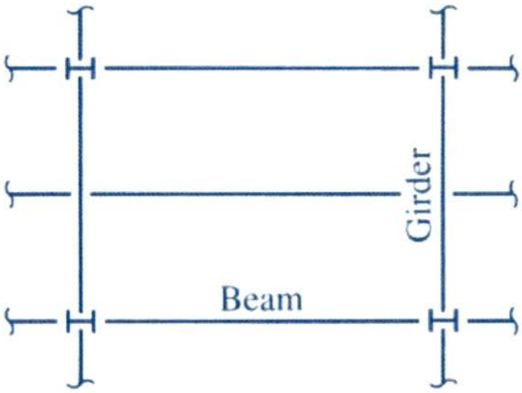
Design Assumptions

- Prismatic beams (straight beams with a uniform cross-section).
- Focus will be on rolled steel sections of W (wide flange) shape and timber beams with rectangular cross-sections.
- Typically the knowns are:
  - Span Length
  - Supporting Conditions
  - Loading
- From these givens, the size and shape of the beam is selected, such that, the maximum flexural stress and maximum shear stress stay within the allowable limits.

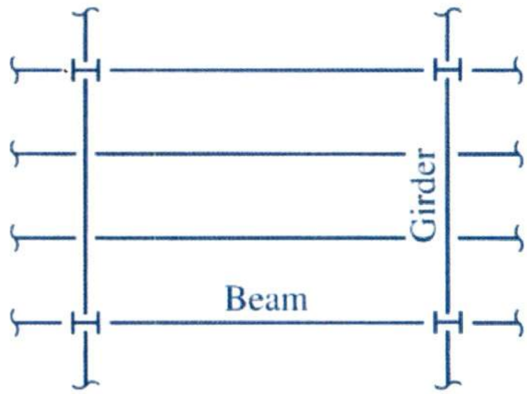
15-2

Basic Considerations in Beam Design

Framing Layout



Center-point Concentration



Third-point Concentration

Loads on the floor supported by the beams can be divided into two major categories:

Dead Load. The dead load consists of the weight of walls, partitions, columns, floors, and roofs. In the design of a beam, the dead load must include an allowance for its own weight.

Live Load. The live load on a floor represents the effect created by occupancy. It includes the weight of humans, furniture, equipment, stored materials, and so on. Building codes provide minimum live loads to be used in the design of various types of buildings. Table 15-1 lists a few typical values.

TABLE 15–1 Minimum Live Loads

Description	*Uniform Live Load* (psf)	(kN/m²)
Warehouse (heavy)	250	12.0
Warehouse (light)	125	6.0
Heavy manufacturing	125	6.0
Light manufacturing	75	3.6
Wholesale stores	100	4.8
Retail stores	75	3.6
Offices	50	2.4
Residential	40	1.9

15-3
Design of Steel Beams

Allowable Stress Design (ASD)

- Allowable stress used in design is often specified by the design code.
- Frequently used code is the American Institute of Steel Construction (AISC) specifications.

AISC prescribes the following allowable values:

Allowable Flexural Stress ➔ $\sigma_{allow} = 0.66 \times \sigma_y$

Allowable Shear Stress ➔ $\tau_{allow} = 0.4 \times \sigma_y$

σ_y = yield strength (from stress & strain diagram)

Most commonly used structural steel is designated by A36, the yield strength σ_y is 36 ksi.

For A36 Steel:

$\sigma_{allow} = 0.66 \times \sigma_y = 0.66 \times 36$ ksi = 24 ksi

$\tau_{allow} = 0.4 \times \sigma_y = 0.4 \times 36$ ksi = 14.5 ksi

Steps for Steel Beam Design

Step 1: Determine the beam span, support conditions, allowable stresses, and other design limitations. Identify or compute the loads.

Step 2: Determine the maximum shear force and the maximum bending moment along the beam. For simple loadings, use the formulas from Table 13-1.

Step 3: Using the maximum value of the bending moment, regardless of the sign, compute the minimum required section modulus from the flexure formula:

$$S_{req} = \frac{M_{max}}{\sigma_{allow}}$$

Step 4: Scan the tables in the appendix, Table A-1, and list several possible choices of W shape that have a section modulus (about the strong axis) greater than the required value computed in step 3. Select the one with the lightest weight per unit length as a trial section (it may not have the smallest section modulus S). Make sure that this section has a value of S slightly greater than S_{req} so that the beam can resist the additional moment produced by the weight of the beam.

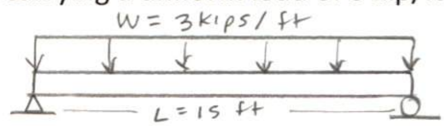
Step 5: The beam selected is checked for shear stress by using the average web shear formula, Equation 14-13:

$$\tau_{avg} = \frac{V_{max}}{dt_w}$$

This calculated stress is usually much lower than T_{allow} ,so shear stress rarely dictates the selection of the size of a steel beam. And the effect of the weight of the beam on the shear stress can usually be neglected.

Example 15-1

Select the lightest W shape steel beam for a 15-ft simple span carrying a uniform load of 3 kip/ft. The beam is supported laterally for its entire length. Use A36 steel.



Solution.

Step 1. Knowns

$w = 3$ kips/ft

$L = 15$ ft

A36 steel

$\sigma_{allow} = 24$ ksi

$\tau_{allow} = 14.5$ ksi

Step 2. Determine V_{MAX} & M_{MAX}

Table 13-1, case 4

$$V_{MAX} = \frac{wL}{2} = \frac{3\text{ kips/ft}\,(15\text{ ft})}{2} = 22.5\text{ kips}$$

$$M_{MAX} = \frac{wL^2}{8} = \frac{3\text{ kip/ft}\,(15\text{ ft})^2}{8} = 84.4\text{ kip}\cdot\text{ft}\left(\frac{12\text{ in}}{\text{ft}}\right) = 1013\text{ kip}\cdot\text{in}$$

Step 3. Calculate S_{req}

$$S_{req} = \frac{M_{MAX}}{\sigma_{allow}} = \frac{1013\text{ kip}\cdot\text{in}}{24\text{ ksi}} = 42.2\text{ in.}^3$$

Step 4. Find W shapes (lightest)

Table A-1(a)

W 14 × 34 $S = 48.6\text{ in.}^3$ (Lightest)

W 12 × 35 $S = 45.6\text{ in.}^3$

W 10 × 45 $S = 49.1\text{ in.}^3$

Select W 14×34 as a trial Section

$$\frac{\text{Beam Weight}}{\text{Load}} = \frac{34\text{ lb/ft}}{3000\text{ lb/ft}} = 0.011 = 1.1\%$$

$$\frac{\text{Extra } S}{S_{req}} = \frac{48.6\text{ in.}^3 - 42.2\text{ in}^3}{42.2\text{ in.}^3} = 0.152 = 15.2\% > 1.1\%$$

∴ satisfactory for bending

Step 5. Check Shear Stress

$d = 13.98$ in., $t_w = 0.285$ in.

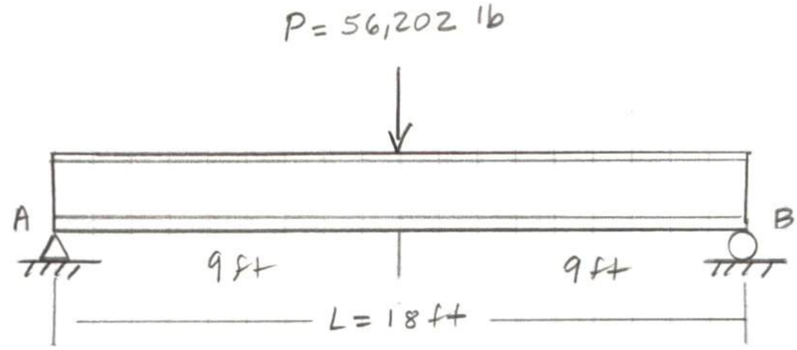
$$\tau_{AVG} = \frac{V_{MAX}}{d\,t_w} = \frac{22.5\text{ kips}}{(13.98\text{ in})(0.285\text{ in})} = 5.65\text{ ksi} < 14.5\text{ ksi}\ ✓$$

USE, W 14×34

Example 15-2 [U.S. System of Units]

A simply supported girder has a span of 18 ft with a concentrated load of 56,202 lb at the midspan. The girder is braced laterally throughout its length. Select the lightest W shape to carry the load for A36 steel.

Solution.



Step 1.

$P = 56{,}202$ lb

$L = 18$ ft

A36 Steel

$\sigma_{allow} = 24$ ksi $\tau_{allow} = 14.5$ ksi

Step 2. Table 13-1, Case 1

$$V_{MAX} = \frac{P}{2} = \frac{56202 \text{ lb}}{2} = 28{,}101 \text{ lb}$$

$$M_{MAX} = \frac{PL}{4} = \frac{56202 \text{ lb} (18 \text{ ft})}{4} = 252{,}909 \text{ lb·ft}$$

Step 3.

$$S_{req} = \frac{M_{MAX}}{\sigma_{allow}} = \frac{252{,}909 \text{ lb·ft} \left(\frac{12 \text{ in}}{\text{ft}}\right)}{24000 \text{ lb/in}^2} = 126.45 \text{ in.}^3 = 127 \text{ in.}^3$$

Step 4. Table A-1(a) Sections with $S > S_{req}$

W 14 × 90 $S = 143$ in.^{3}

W 16 × 89 $S = 155$ in.^{3}

W 18 × 97 $S = 188$ in.^{3}

Select W 16 × 89 as a trial section (lightest)

(uniform load) $$M_{WT} = \frac{wL^2}{8} = \frac{(89 \text{ lb/ft})(18 \text{ ft})^2}{8} = 3605 \text{ lb·ft}$$

$$\frac{M_{WT}}{M_{MAX}} = \frac{3605 \text{ lb·ft}}{252{,}909 \text{ lb·ft}} = 0.0143 = 1.4\%$$

$$\frac{\text{Extra } S}{S_{req}} = \frac{155 \text{ in}^3 - 127 \text{ in.}^3}{127 \text{ in.}^3} = 0.22 = 22\% > 1.4\%$$

Beam Satisfactory for bending

Step 5. Check Shear Stress

W 16 × 89

$d = 16.75$

$t_w = 0.525$ in

$$\tau_{AVG} = \frac{V_{MAX}}{d\,t_w} = \frac{28{,}101 \text{ lb}}{(16.75 \text{ in})(0.525 \text{ in})} = 3.2 \text{ ksi} < 14.5 \text{ ksi}$$

✓ bending

use, W 16 × 89